Chapter 1

Linear Algebra

1.1 Basic Stipulations

a linear space: two sets, three operations.
$\{\mathbb{X}, +\}$ is a Abelian group;
$\{A, +, *\}$ is a field;
$\times : A \times \mathbb{X} \to \mathbb{X}$.
($\times : Z \times \mathbb{X} \to \mathbb{X}$ has implicated such an object.)
3 operations: $+$, *, $\times$. if $\{x, y\} \subset \mathbb{X}, \{\alpha, \beta\} \subset A$, then we have 3 combinations of these operations:

$$(\alpha + \beta) \times x = (\alpha \times x) + (\beta \times x)$$

$$(\alpha * \beta) \times x = \alpha * (\beta \times x)$$

$$\alpha \times (x + y) = (\alpha \times x) + (\beta \times y)$$

then, we want to know what constructs and properties could such an object (linear space) possess.

1.2 Basic Structure

group and field

dimension and subspace
for 0 of group $\{\mathbb{X}, +\}$, what does $x + y +, ..., +z = 0$ means?
$\forall x \in \mathbb{X}, 0 * x = 0, 0 \in A, 0 \in \mathbb{X}$;
$\forall \alpha \in A, \alpha * 0 = 0, 0 \in \mathbb{X}$;
if $\alpha \neq 0, x \neq 0$,then $\alpha * x \neq 0$;
but, $\alpha * x + \beta * y = 0$ is possible! when $\exists \alpha, \beta, x, y \neq 0$.
then, what does that means?
means that

Lemma 1.2.1 $\forall\gamma \in A, \{z|z = \gamma * x\}$ *is a linear space; any such* $y \in \{z\}$*;* x *and* y *is symmetry in the lemma;* $\{Z\}$ *is a subgroup of* $\mathbb{X}$*.*

if $\exists$ y is not $\in \{z\}$, then $\forall\alpha, \beta \neq 0, \alpha * x + \beta * y \neq 0$, at this situation, $\alpha * x + \beta * y + \gamma * z = 0$ is possible!
what does this means?
means that:

Lemma 1.2.2 $\forall\alpha, \beta \in A, \{z|z = \alpha * x + \beta * y\}$ *is a linear space;* y *and* z *is symmetry in this lemma; and* $\{z\}$ *is a subgroup of* $\mathbb{X}$*.*

let's go on! if $\{z\} = \mathbb{X}$, and z can be expressed as

$$z = \alpha * x + ... + \beta * y$$

then we can say $\{x, ..., y\}$ is bases of $\mathbb{X}$. the number of the elements of $\{x, ..., y\}$ is the dimension number of $\mathbb{X}$.
because of the form $x+y+, ..., +z = 0$ can be used to generate all the space $\mathbb{X}$, we name such $\{x, y, ...z\}$ is linear dependent. and if $z = x + ... + y$, we name $x + ... + y$ is a linear combination of z.

Theorem 1.2.3 *any vector* z *of* $\mathbb{X}$ *can be expressed as its bases's unique linear combination.*

coordinate transform
if $\mathbb{X}$ is a n-dimension linear space, then any non-linear dependent n elements of $\mathbb{X}$ can be used as its base.

isomorphism
How to retain the structure of a linear space?