Chapter 1

Manifold

1.1 Basic Notions

When we mention Euclid space, maybe it induct excess meanings. So, we can try to construct an object that its meaning is less than Euclid space but however, is meaningful and analyzable.

At least, we need some points, then, if we want a point is analyzable, it must has its neighborhood, moreover, to make it computable or realizable, let its neighborhood homeomorphism with an open set of Euclid space.

Such points constitutes a set, then make this set is a separated set, a Hausdorff set, we definite such a set a **manifold**.

The point $x \in M$, M is a manifold. U is a neighborhood of x, and there is a homeomorphism $\phi : U \mapsto \phi(U)$. then, Any such a $(U, \phi(U))$ means a realization of any point of U: $\phi(y)$ is the coordination of $y \in U$.

So, we definite $(U, \phi(U))$ a **map chart** of M.

Because any point of a manifold has its neighborhood as well as an homeomorphism, then for any pair of points (x, y), their neighborhood (U, V) have intersection or have not:

i) if $U \cap V \neq \theta$, then $\phi(U) \circ \varphi(V)^{-1}$ and $\varphi(V) \circ \phi(U)^{-1}$ are two real functions from an open set to another of Euclid space. we can treat them as coordinations transformation, we can let them be C^r.

ii) if $U \cap V = \theta$, then nothing.

When i) and ii) are satisfied, we name U and V are C^r consistent.

If we want to concern the movement of point on a manifold, and to analyze such movement, we'd better let the functions $\phi(U) \circ \varphi(V)^{-1}$ and $\varphi(V) \circ \phi(U)^{-1}$ have good enough analytical property, because their analytical property restrict the analyzable of the manifold.

Then, as soon as we get enough map charts $(U, \phi(U))$ of manifold M, we can analyze the point movement on it. the "enough" means:

i) $\mathbb{A} = \{U, V, W, ...\}$ is a open cover of M;

ii) Any two of $\mathbb{A}$ are C^r consistent;

iii) $\mathbb{A}$ is the maximum.

then, we name $\mathbb{A}$ a C^r differential structure of M, correspondingly, M is a C^r differential manifold.

Let's treat some familiar set as manifold, and to see how does adding various differential structures on them means.

Example Euclid space R^n

Example Sphere S^n

Example Projective space P^n

Example Milnor monstrous ball.